Semigroup Forum (2021) 102:568–574
https://doi.org/10.1007/s00233-021-10166-1

SHORT NOTE

Polynomial central set theorem near zero

Aninda Chakraborty[2] · Sayan Goswami[1]

Received: 18 December 2019 / Accepted: 19 December 2020 / Published online: 23 February 2021

Abstract
Hindman and Leader introduced the set 0^+ of ultrafilters on $(0, 1)$, characterized the smallest ideal of $\left(0^+, +\right)$ and proved the Central Set Theorem near 0. Recently Polynomial Central Set Theorem has been proved by Bergelson, Johnson Jr. and Moreira. In this note, we will Polynomial Central Set Theorem near 0.

Keywords Van der Waerden's theorem · Ramsey theory near zero · Polynomial Hales–Jewett theorem

1 Introduction

Given a discrete semigroup $(S, \cdot)$, it is well known that one can extend the operation $\cdot$ to βS, the Stone-Čech compactification of S so that $(\beta S, \cdot)$ is a right topological semigroup (i.e., for each $p \in \beta S$, the function $\rho_p : \beta S \to \beta S$, defined by $\rho_p(q) = q \cdot p$, is continuous) with S contained in the topological center (i.e., for each $x \in S$, the function $\lambda_x : \beta S \to \beta S$, defined by $\lambda_x(p) = x \cdot p$, is continuous). Further, this operation has frequently proved to be useful in Ramsey Theory. See [7] for an elementary introduction to the semigroup $(\beta S, \cdot)$ and its combinatorial applications.

It is also well known that if S is not discrete, such an extension may not be possible (see [6, Section 2] where it is shown how bad the situation is for any dense subsemigroup of $([0, \infty], +)$).

Surprisingly, however, it has turned out to be possible to use the algebraic structure of $\beta\mathbb{R}_d$ to obtain Ramsey Theoretic results that are stated in terms of the usual topology

Communicated by Jimmie D. Lawson.

✉ Sayan Goswami
sayan92m@gmail.com

Aninda Chakraborty
anindachakraborty2@gmail.com

1 Department of Mathematics, University of Kalyani, Kalyani, Nadia, West Bengal 741235, India

2 Department of Mathematics, Government General Degree College at Chapra, Chapra, Nadia, West Bengal 741123, India

RICHNESS OF ARITHMETIC PROGRESSIONS IN COMMUTATIVE SEMIGROUPS

Aninda Chakraborty
Government General Degree College at Chapra, Chapra, West Bengal, India
anindachakraborty2@gmail.com

Sayan Goswami
Department of Mathematics, University of Kalyani, Kalyani, West Bengal, India
sayan92m@gmail.com

Received: 4/23/19, Revised: 1/2/20, Accepted: 3/14/20, Published: 4/13/20

Abstract

Furstenberg and Glasner proved that for an arbitrary $k \in \mathbb{N}$, any piecewise syndetic set contains k-term arithmetic progressions and, in a sense to be made precise later, the set of such arithmetic progressions is piecewise syndetic in $\mathbb{Z}^2$. They used the algebraic structure of $\beta\mathbb{N}$. The above result was extended for arbitrary semigroups by Bergelson and Hindman, again using the structure of the Stone-Čech compactification of a general semigroup. Beiglböck provided an elementary proof of the above result and asked whether the combinatorial argument in his proof can be enhanced in a way which makes it applicable to a more abstract setting. In a recent work S. Jana and the second author of this paper provided an affirmative answer to Beiglböck's question for countable commutative semigroups. In this work we will extend the result of Beiglböck in different types of settings.

1. Introduction

A subset S of $\mathbb{Z}$ is called syndetic if there exists $r \in \mathbb{N}$ such that $\bigcup_{i=1}^{r}(S-i) = \mathbb{Z}$ and it is called thick if it contains arbitrary long intervals. Sets which can be expressed as the intersection of thick and syndetic sets are called piecewise syndetic sets.

For a general commutative semigroup $(S,+)$, a set $A \subseteq S$ is said to be syndetic in $(S,+)$, if there exists a finite nonempty set $F \subseteq S$ such that $\bigcup_{t\in F} -t + A = S$ where $-t + A = \{s \in S : t + s \in A\}$. A set $A \subseteq S$ is said to be thick if for every finite nonempty set $E \subseteq S$, there exists an element $x \in S$ such that $E + x \subseteq A$. A set $A \subseteq S$ is said to be piecewise syndetic if there exists a finite nonempty set $F \subseteq S$ such that $\bigcup_{t\in F}(-t + A)$ is thick in S. It can be proved that a piecewise syndetic set is the intersection of a thick set and a syndetic set [10, Theorem 4.49].

European Journal of Combinatorics 104 (2022) 103539

Contents lists available at ScienceDirect

European Journal of Combinatorics

journal homepage: www.elsevier.com/locate/ejc

Hales–Jewett type configurations in small sets

Aninda Chakraborty [a], Sayan Goswami [b]

[a] Department of Mathematics, Government General Degree College at Chapra, Chapra, Nadia, West Bengal, India
[b] Department of Mathematics, University of Kalyani, Kalyani-741235, Nadia, West Bengal, India

ARTICLE INFO

Article history:
Received 23 September 2020
Accepted 16 March 2022
Available online xxxx

ABSTRACT

In a recent work, N. Hindman, D. Strauss and L. Zamboni have shown that the Hales–Jewett theorem can be combined with a sufficiently well behaved homomorphisms. Their work was completely algebraic in nature, where they used the algebra of Stone–Čech compactification of discrete semigroups. They proved the existence of those configurations in piecewise syndetic sets, which are Ramsey theoretic rich sets. In our work we will show those forms are still present in very small but Ramsey theoretic sets, (like J-sets, C-sets) and our proof is purely elementary in nature.

1. Introduction

Let $\omega = \mathbb{N} \cup \{0\}$, where $\mathbb{N}$ is the set of positive integers. Then ω is the first infinite ordinal. For any set X, let $\mathcal{P}_f(X)$ be the set of all nonempty finite subsets of X.

Given a nonempty set $\mathbb{A}$ (or alphabet) we let S_0 be the set of all finite words $w = a_1a_2\ldots a_n$ with $n \geq 1$ and $a_i \in \mathbb{A}$. The quantity n is called the length of w and denoted $|w|$. The set S_0 is naturally a semigroup under the operation of concatenation of words. We will denote the empty word by θ. For each $u \in S_0$ and $a \in \mathbb{A}$, we let $|u|_a$ be the number of occurrences of a in u. We will identify the elements of $\mathbb{A}$ with the length-one words over $\mathbb{A}$.

Let v (a variable) be a letter not belonging to $\mathbb{A}$. By a variable word over $\mathbb{A}$ we mean a word w over $\mathbb{A} \cup \{v\}$ with $|w|_v \geq 1$. We let S_1 be the set of variable words over $\mathbb{A}$. If $w \in S_1$ and $a \in \mathbb{A}$, then $w(a) \in S_0$ is the result of replacing each occurrence of v by a.

A finite coloring of a set A is a function from A to a finite set $\{1, 2, \ldots, n\}$. A subset B of A is monochromatic if the function is constant on B. If $\mathbb{A}$ is any finite nonempty set and S is the free

E-mail addresses: anindachakraborty2@gmail.com (A. Chakraborty), sayan92m@gmail.com (S. Goswami).

https://doi.org/10.1016/j.ejc.2022.103539
0195-6698/